Keio University

# Circuit Design for Measurement-Based Quantum Carry Lookahead Adder

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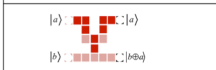
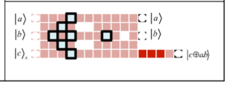
## Introduction : VBE MBQCLA

Raussendorf et. al, Phys. Rev. A 68, 022312 (2003)

- Quantum information propagation in a cluster is driven by the pattern of measurement bases, regardless of the measurement outcomes.
- By subsequent single-qubit measurements, quantum gates are effected on the logical qubits encoded in the cluster state.

| | |
|---|---|
| | CNOT |
| 1 |  |
| | Toffoli Phase Gate |
| 2 |  |

Barenco et. al, Phys. Rev. A 52, 3457 - 3467 (1995)

Resources $\approx 304n$

Depth $\approx 3n$

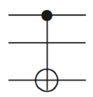
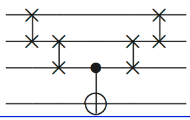
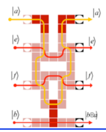
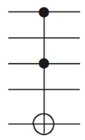
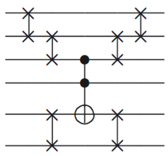
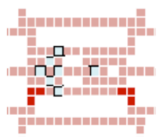
**Purpose**

**to present the design for the MBQCLA and evaluate it in terms of its execution speed and resources requirements.**



## Space and depth tradeoffs between abstract and measurement-based quantum computation in nearest-neighbor architectures

Barenco et. al, Phys. Rev. A 52, 3457 - 3467 (1995) Raussendorf et. al, Phys. Rev. A 68, 022312 (2003)

| Quantum gates | Abstract | Measurement-based |
|---|---|---|
|  |  |  |
|  |  |  |



| Quantum gates | Abstract | | Measurement-based | |
|---|---|---|---|---|
| CNOT | Resources | $2+n$ qubits<br>$2n$ SWAP gates | Resources | $15+18n$ qubits |
| | Depth | $2n+1$ | Depth | 1 |
| Toffoli | Resources | $3+n_c+n_t$ qubits<br>$2(n_c+n_t)$ SWAP gates | Resources | $54+31(n_c+n_t)$ qubits |
| | Depth | $2(n_c+n_t)+1$ | Depth | 2 |



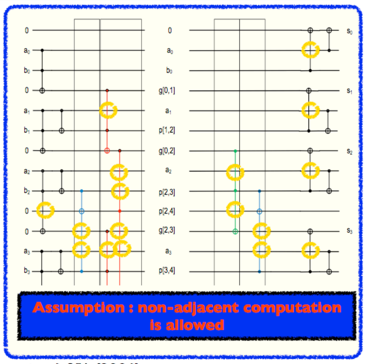
## Comparison between abstract and measurement-based quantum computation in nearest-neighbor architectures

| | Abstract | Measurement-based |
|---|---|---|
| Depth | $f(n)$ | Constant |
| Resources | Cost less resources | Cost more resources |
| Physical implementations | Physically unrealistic | Physically realistic |



## Logarithmic quantum carry-lookahead adder : out-of-place and in-place

- Depth : $\Theta(log\ (n))$
- The out-of-place form of QCLA performs $|a, b, 0\rangle \longrightarrow |a, b, a+b\rangle$
- The in-place form of QCLA performs $|a, b\rangle \longrightarrow |a, a+b\rangle$




Draper et. al, Quantum Information & Computation 6, 351 (2006).

## Evaluating Algorithms Executed using MBQC

$$Size \approx \sum_i (\mathcal{V}(n) \times \mathcal{B}_i \times \mathcal{T}_i + \mathcal{X}_i\ (\mathcal{R}_i - \mathcal{E}_i \times \mathcal{T}_i)) + \mathcal{L}_{SWAP} \times \mathcal{S}_{SWAP}$$

- $i \in$ {Toffoli Phase, CNOT, and NOT Gates}
- $\mathcal{V}(n)$ = number of logical qubits
- $\mathcal{B}_i$ = number of rounds for the gate of type $i$
- $\mathcal{X}_i$ = number of gates of type $i$
- $\mathcal{T}_i$ = width of the $i$-gate (in lattice sites)
- $\mathcal{R}_i$ = number of lattice qubits in an $i$-gate
- $\mathcal{E}_i$ = number of logical qubits in an $i$-gate (generally, one to three)
- $\mathcal{L}_{SWAP}$ is number of lattice qubits in a cluster state SWAP gate
- $\mathcal{S}_{SWAP}$ = number of SWAP gates in a QCLA circuit, which is dependent on the mapping of the logical qubits to positions on the lattice.

**The optimal circuit resources**

$$\sum_{i \in Quantum\ Gate} \mathcal{X}_i \mathcal{R}_i$$

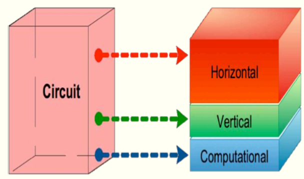



Illustration for circuit costs for MBQCLA. On a two-dimensional Manhattan grid, the costs contain computational and communication costs. The resources for communication costs can be separated into two types of resources: horizontal (wires) and vertical (SWAP gates).



## Requirements and Performance of the out-of-place MBQCLA

| Parameter | Value |
|---|---|
| Pitch | 4 |
| Variables (Logical Qubits) | $\mathcal{V}(n) = 4n$-$\lfloor \log_2 n \rfloor$+1 |
| Width | $\mathcal{W}idth(n)$=15×($\lfloor\log_2(n)\rfloor$+$\lfloor\log_2(n/3)\rfloor$)+85 |
| Height | $\mathcal{H}eight(n)$=4×(4$n$-$w(n)$-$\lfloor\log_2 n\rfloor$+1)-3 |
| Area | $\mathcal{H}eight(n)$×$\mathcal{W}idth(n)$ = (4×(4$n$-$w(n)$-$\lfloor\log_2 n\rfloor$+1)-3)× (14×($\lfloor\log_2(n)\rfloor$+$\lfloor\log_2(n/3)\rfloor$)+85) |
| Number of Clustering Operations | (4$n$-$w(n)$-$\lfloor\log_2 n\rfloor$+1)× (15×$\lfloor\log_2(n)\rfloor$+$\lfloor\log_2(n/3)\rfloor$+85)-1)+ (15×$\lfloor\log_2(n)\rfloor$+$\lfloor\log_2(n/3)\rfloor$+85))× (4$n$-$w(n)$-$\lfloor\log_2 n\rfloor$) |
| Circuit Depth | $\lfloor\log_2(n)\rfloor$+$\lfloor\log_2(n/3)\rfloor$+7 |
| Size (Number of Qubits) | $-3271+899n-419w(n)-377\lfloor\log_2(n)\rfloor+56n\lfloor\log_2(2n/3)\rfloor-14w(n)\lfloor\log_2(2n/3)\rfloor$ $+42n\lfloor\log_2(n)\rfloor+168n\lfloor\log_2(n)\rfloor-14\lfloor\log_2(2n/3)\rfloor\lfloor\log_2(n)\rfloor-42\lfloor\log_2(n)\rfloor^2$ $+6\times(2n+\sum_{t_p=1}^{logn-1} 2(n-2^{t_p})+\sum_{t_g=1}^{logn} 2(2(\lfloor log(n)\rfloor)+ (2^{t_g})(\lfloor logt_g\rfloor)$ $+\sum_{t_c=1}^{\lfloor log\frac{2n}{3}\rfloor} 2(n-\lfloor logt_c\rfloor))$ |

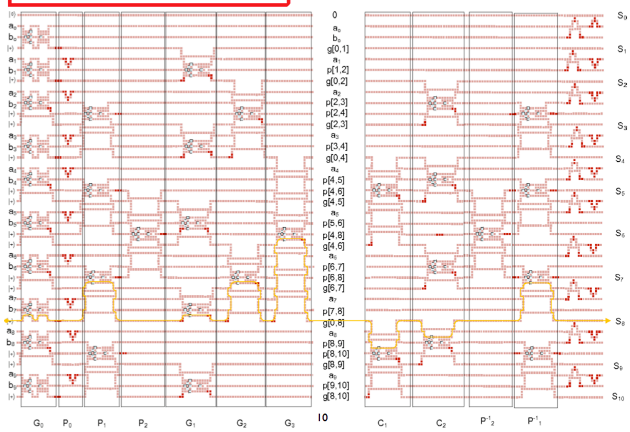


## Requirements and Performance of the in-place MBQCLA

| Parameter | Value |
|---|---|
| Pitch | 4 |
| Variables (Logical Qubits) | $\mathcal{V}(n) = 4n$-$\lfloor \log_2 n \rfloor$+1 |
| Width | $\mathcal{W}idth(n)$=15×($\lfloor\log_2 n\rfloor$+ $\lfloor\log_2(n-1)\rfloor$+ $\lfloor\log_2 \frac{n}{3}\rfloor$+$\lfloor\log_2 \frac{n-1}{3}\rfloor$)+157 |
| Height | $\mathcal{H}eight(n)$=4×(4$n$-$w(n)$-$\lfloor\log_2 n\rfloor$+1)-3 |
| Area | $\mathcal{H}eight(n)$×$\mathcal{W}idth(n)$ = (4×(4$n$-$w(n)$-$\lfloor\log_2 n\rfloor$+1)-3)× 14×($\lfloor\log_2 n\rfloor$+ $\lfloor\log_2(n-1)\rfloor$+ $\lfloor\log_2 \frac{n}{3}\rfloor$+$\lfloor\log_2 \frac{n-1}{3}\rfloor$)+157 |
| Number of Clustering Operations | (4$n$-$w(n)$-$\lfloor\log_2 n\rfloor$+1)× (15×($\lfloor\log_2 n\rfloor$+ $\lfloor\log_2(n-1)\rfloor$+ $\lfloor\log_2 \frac{n}{3}\rfloor$+$\lfloor\log_2 \frac{n-1}{3}\rfloor$)+156)+ (15×($\lfloor\log_2 n\rfloor$+ $\lfloor\log_2(n-1)\rfloor$+ $\lfloor\log_2 \frac{n}{3}\rfloor$+$\lfloor\log_2 \frac{n-1}{3}\rfloor$)+157)× (4$n$-$w(n)$-$\lfloor\log_2 n\rfloor$) |
| Circuit Depth | $\lfloor\log_2 n\rfloor$+ $\lfloor\log_2(n-1)\rfloor$+ $\lfloor\log_2 \frac{n}{3}\rfloor$+$\lfloor\log_2 \frac{n-1}{3}\rfloor$+14 |
| Size (Number of Qubits) | $-3068+2896n-138w(n-1)-162w(n)+16\lfloor\log_2 \frac{n-1}{3}\rfloor$ $+64n\lfloor\log_2 \frac{n-1}{3}\rfloor-146\lfloor\log_2(n\text{-}1)\rfloor$ $+64n\lfloor\log_2(n\text{-}1)\rfloor+16\lfloor\log_2(n/3)\rfloor+64n\lfloor\log_2(n/3)\rfloor$ $+21\lfloor\log_2(n)\rfloor+64n\lfloor\log_2(n)\rfloor$ $-16\lfloor\log_2(n-1)\rfloor\lfloor\log_2(n)\rfloor-16\lfloor\log_2\frac{n}{3}\rfloor\lfloor\log_2(n)\rfloor-16\ \lfloor\log_2(n)\rfloor^2+$ $6\times(4n\text{-}1+\ 4\sum_{t_p=1}^{logn-1} 2(n-2^{t_p})+\ 2\sum_{t_g=1}^{logn} 2(2(\lfloor log(n)\rfloor)+ (2^{t_g})(\lfloor logt_g\rfloor)$ $+\ 2\sum_{t_c=1}^{\lfloor log\frac{2n}{3}\rfloor} 2(n-\lfloor logt_c\rfloor))$ |

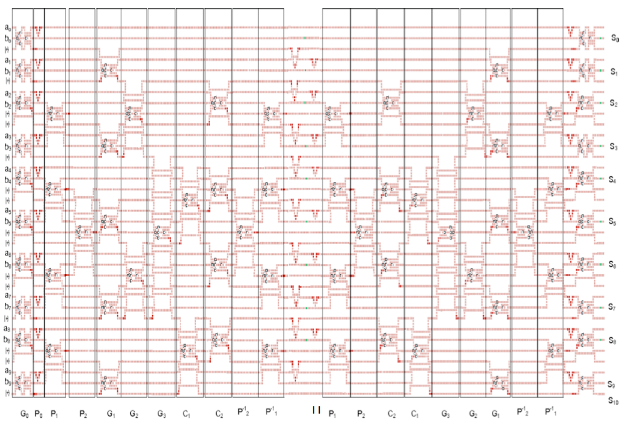


## Out-of-place MBQCLA





## In-place MBQCLA





## Size and depth comparison of various type of MBQCLA circuits

**VBE MBQCLA**

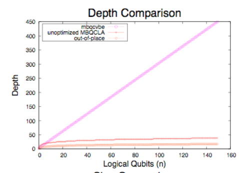
- Resources : $\approx 304n$
- Depth : $\approx 3n$

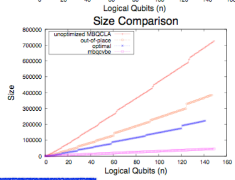
**Out-of-place MBQCLA :**

- Resources :$\approx 901n + 224n \times \lfloor \log_2(n) \rfloor$
- Depth : $\approx 2(\lfloor log\ n \rfloor + \lfloor log\frac{2n}{3} \rfloor) + 13$

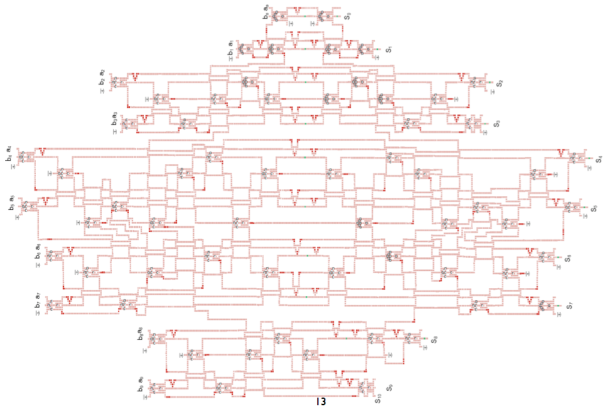
**In-place MBQCLA :**

- Resources : $\approx\ 2896n + 64n\lfloor log(n) \rfloor$
- Depth : $\approx 2(\lfloor log(n) \rfloor + \lfloor log(n-1) \rfloor + \lfloor log\frac{n}{3} \rfloor + \lfloor log\frac{n-1}{3} \rfloor + 14)$



## Manual optimization of MBQCLA circuits



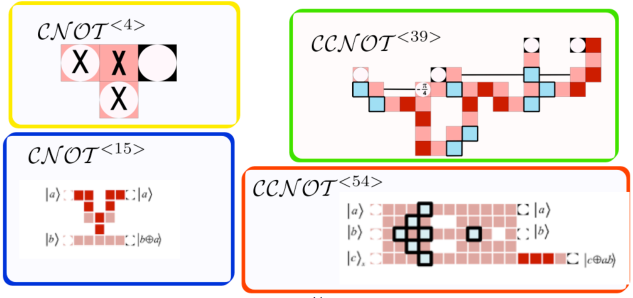


## MBQC formalism

Danos *et. al*, **Journal of ACM**, volume 54, pages 8 (2007), ISSN 0004-5411

$$\mathcal{U}:=(\{\text{Resources}\}_{\mathcal{U}}, \{\text{Input}\}_{\mathcal{U}}, \{\text{Output}\}_{\mathcal{U}}, \{EMC\}_{\mathcal{U}})$$

$$\mathcal{CNOT}^{<4>} := (\{1, 2, 3, 4\}, \{1, 4\}, \{3, 4\}, \{X_4^{s_3} Z_4^{s_2} Z_1^{s_2} M_3^x M_2^x E_{13} E_{23} E_{34}\})$$





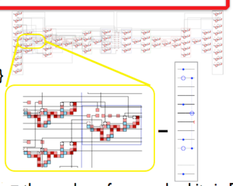

## In-place GSQCLA

The number of entanglement operations in GSQCLA

$$\sum_k E_k \mathcal{X}_k; \quad k \in \{\text{Toffoli Phase, CNOT, and NOT gates}\}$$

$E_k$ = entanglement operations in type *k* gates

$\mathcal{X}_k$ = number of gates of type *k*



The qubit resources after adjustment in GSQCLA

$\sum_m \mathcal{X}_m \mathcal{R}_m - \sum_l \mathcal{N}_l(\mathcal{Q}_P, \mathcal{Q}_G, \mathcal{Q}_C, \mathcal{Q}_{add})$; $\mathcal{Q}_P, \mathcal{Q}_G, \mathcal{Q}_C, \mathcal{Q}_{add}$ = the number of removed qubits in P, G, C , and additional rounds in the circuit, respectively

$\mathcal{N}_l$ = number of removed qubits of type *l* circuit

$\mathcal{Q}_P = \sum_{t_p=1}^{\lfloor log(n)\rfloor-1} 2(\lfloor \frac{n}{2^{t_p}} \rfloor - 1)$; $\mathcal{Q}_C = \sum_{t_c=1}^{\lfloor log(\frac{2n}{3})\rfloor-1} 3\lfloor \frac{(n-2^{t_c-1})}{2^{t_c}} \rfloor$; $\mathcal{Q}_C = \sum_{t_c=1}^{\lfloor log(\frac{2n}{3})\rfloor-1} 3\lfloor \frac{(n-2^{t_c-1})}{2^{t_c}} \rfloor$

$\mathcal{Q}_{add}$ varies depending on the type of circuit

The number of entanglement operations for the in-place GSQCLA

$$444n - 129\,(w(n) - w(n-1) - \lfloor log\ n \rfloor - \lfloor log\ (n-1) \rfloor) - 318.$$

The number of qubits for the in-place GSQCLA :

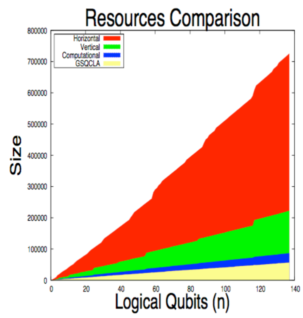
$$410n - 117\,(w(n) - w(n-1) - \lfloor log\ n \rfloor - \lfloor log\ (n-1) \rfloor) - 4 \sum_{t_p=1}^{\lfloor log(n)\rfloor-1} 2(\lfloor \frac{n}{2^{t_p}} \rfloor - 1)$$

$$-2 \sum_{t_g=1}^{\lfloor log(n)\rfloor} 3\lfloor \frac{n}{2^{t_g}} \rfloor - 2 \sum_{t_c=1}^{\lfloor log(\frac{2n}{3})\rfloor-1} 3\lfloor \frac{(n-2^{t_c-1})}{2^{t_c}} \rfloor - 261.$$



## Resources comparison of various forms of in-place MBQC

- The horizontal communication costs of MBQCLA $\approx 77\ \%$
- The circuit vertical communication $\approx 11\ \%$
- The cost of the computational circuit $\approx 12\ \%$
- The in-place GSQCLA $\approx 9\ \%$






## Conclusion

1. The circuit designs and depth-space resources tradeoffs for several forms of a measurement-based quantum carry-lookahead adder (MBQCLA) and graph-state quantum carry-lookahead adder (GSQCLA) are presented.
2. By bending the network and removing the border between the rounds, the optimization of the in-place MBQCLA circuit changes its shape from rectangle to diamond-like form.

**Thank you very much for your attention**